

# Biotechnological processes: calculation exercises

Dr. Jan B. A. Arends

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# General

- Questions: [jan.arends@ugent.be](mailto:jan.arends@ugent.be)
  - Case studies: contact Marlies Christiaens  
[marlies.christiaens@ugent.be](mailto:marlies.christiaens@ugent.be)
- ➔ Put [Biotech] in the subject for any email concerning the course
- Course notes: VLK cursusdienst
  - Subject first treated in theoretical course, so exercise sessions are complementary with theory
  - New course notes: small errors possible!

# General

- Goal: getting insight in biotechnological processes in wastewater treatment by **making calculations**
- Actively making exercises during courses, no passive listening
- Start with short theoretical background, then making exercises yourself !

# When do we meet

Oct 29<sup>th</sup>

Stoichiometry

Nov 5<sup>th</sup>

Removal of organic matter  
Sludge treatment

13h00  
Denduyver

Nov 12<sup>th</sup>

Nutrient removal  
Sedimentation

13h00  
Denduyver

Dec 2<sup>nd</sup>

Combined exercise // Overall design  
8h30  
HILO auditorium A

*Filters, aeration, fluidized-bed, oxidation ditch: additional information*

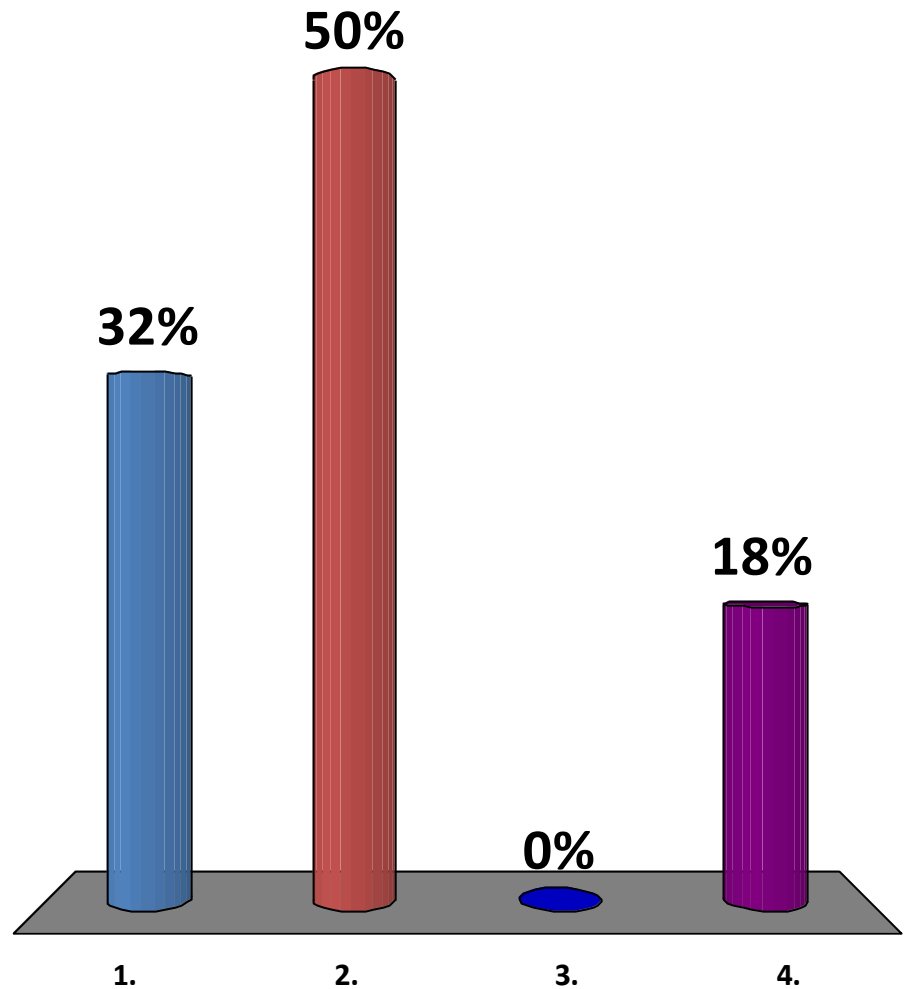
# Part 2: Stoichiometry

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# Chose your option

1. ManaMa  
Milieusanering
2. CES&T (IMENVI)
3. TIWM
4. Other



Stoichiometry: Balancing of reactions

Thermodynamics: calculation of Gibbs free energy change of a reaction

# Wrap-up

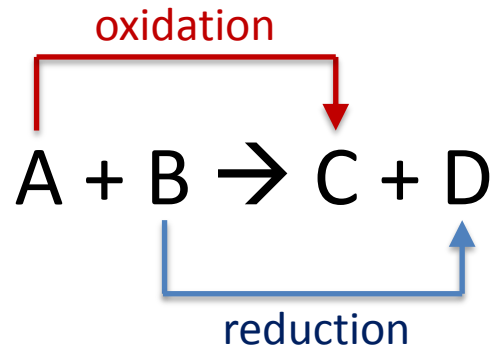
At the end of the semester you can:

- Calculate the COD of simple molecules used in WWTP
- Determine the energy gain/input for biochemical reactions using the growth reference system (GRS)
- Comment on sludge production under various conditions in WWTP



# Growth reference system (GRS)

- Biochemical processes: Redox reactions



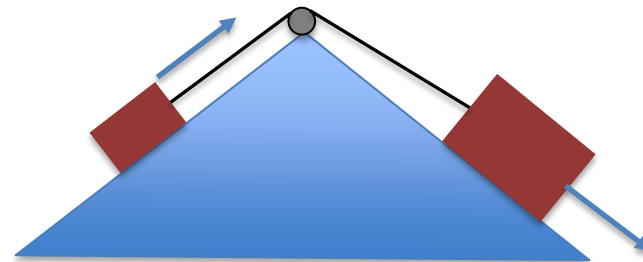
→ Thermodynamics: thermodynamically feasible

**Driving force :  $\Delta G'^0$**

$\Delta G'^0 < 0$



$\Delta G'^0 > 0$



Different references  
to calculate  $\Delta G'^0$

Standard conditions (1 bar, 298.15 K, 1 mol/L, pH = 0)

BIOCHEMICAL Standard conditions (1 atm, 298.15 K, 1 mol/L, pH = 7)

# Growth reference system (GRS)

- $\Delta G_r^{\prime 0} = \sum (v_x \cdot \gamma_x \cdot \Delta G_{e,x})$ 
    - $v_x$  = stoichiometric coef.  $v_x > 0$  for products
    - $\gamma_x$  = degree of reduction  $v_x < 0$  for reagents
    - $\Delta G_{e,x}$  = Gibbs free energy of formation (kJ/mol e<sup>-</sup>) (Table 2)
  - Gibbs energy of formation is 0 for stable end products in **microbial** systems
    - HCO<sub>3</sub><sup>-</sup>, SO<sub>4</sub><sup>2-</sup>, NO<sub>3</sub><sup>-</sup>, H<sub>2</sub>O, H<sup>+</sup>
  - At biochemical standard conditions
    - 1 atm, 298.15K, pH 7, 1 mol/L
- ➔ Never mix these values with conventional thermodynamical data!!!!!!**

# Degree of reduction ( $\gamma$ )

- See also table 1

– C +4

– H +1

– O -2

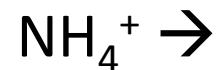
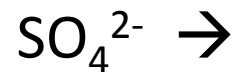
– S +6

– P +5

– N +5

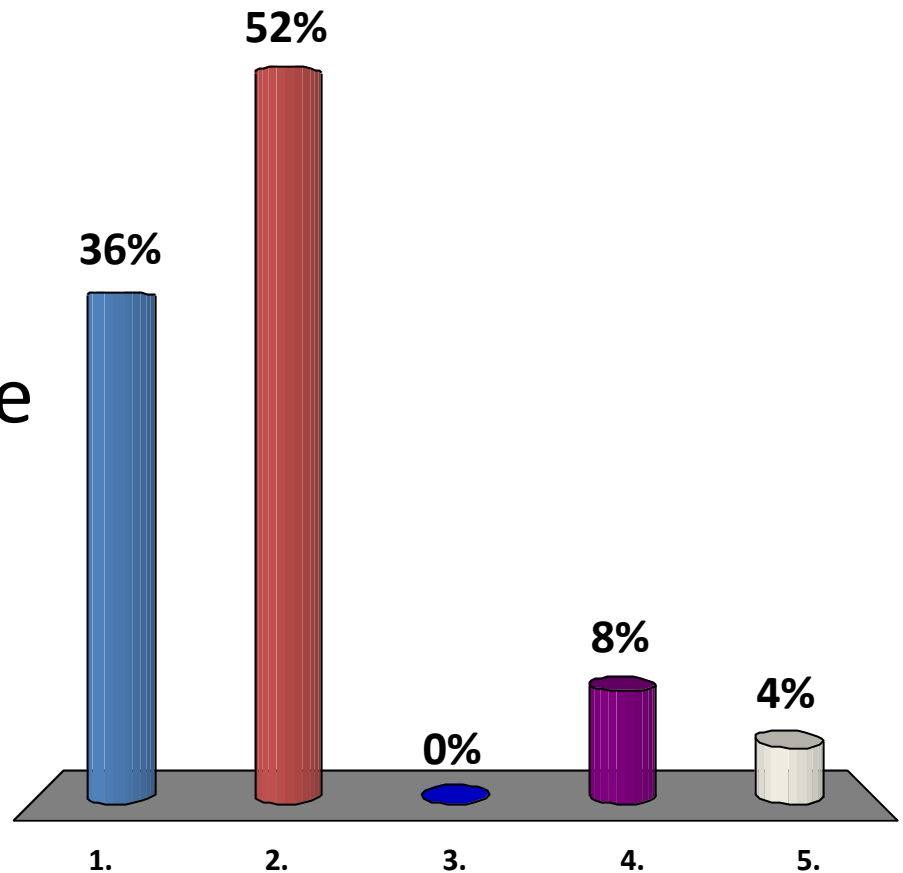
I	II	III	IV	V	VI	VII	VIII
hydrogen 1 <b>H</b> 1.0079							helium 2 <b>He</b> 4.0026
lithium 3 <b>Li</b> 6.941	beryllium 4 <b>Be</b> 9.0122	boron 5 <b>B</b> 10.811	carbon 6 <b>C</b> 12.011	nitrogen 7 <b>N</b> 14.007	oxygen 8 <b>O</b> 15.999	fluorine 9 <b>F</b> 18.998	neon 10 <b>Ne</b> 20.180
sodium 11 <b>Na</b> 22.990	magnesium 12 <b>Mg</b> 24.305	aluminum 13 <b>Al</b> 26.982	silicon 14 <b>Si</b> 28.086	phosphorus 15 <b>P</b> 30.974	sulfur 16 <b>S</b> 32.065	chlorine 17 <b>Cl</b> 35.453	argon 18 <b>Ar</b> 39.948

→ Degree of reduction = sum of reduction of all elements in a molecule



# What is the degree of reduction of: Sulfate and ammonium?

1.  $\text{SO}_4^{2-} = -2$  &  $\text{NH}_4^+ = 1$
- ✓ 2.  $\text{SO}_4^{2-} = 0$  &  $\text{NH}_4^+ = 8$
3.  $\text{SO}_4^{2-} = 4$  &  $\text{NH}_4^+ = 4$
4. I don't know?
5. My answer is not here



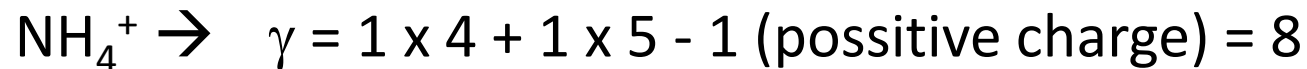
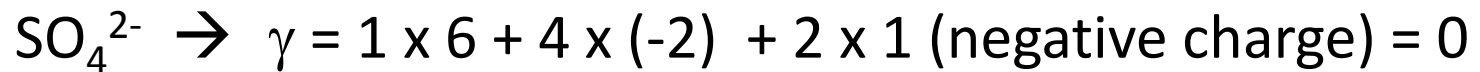
# Degree of reduction ( $\gamma$ )

- See also table 1

– C	+4
– H	+1
– O	-2
– S	+6
– P	+5
– N	+5

I		II		III		IV		V		VI		VII		VIII	
hydrogen 1 H 1.0079														helium 2 He 4.0026	
	beryllium 4 Be 9.0122													neon 10 Ne 20.180	
		boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998									
		aluminum 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453								argon 18 Ar 39.948	

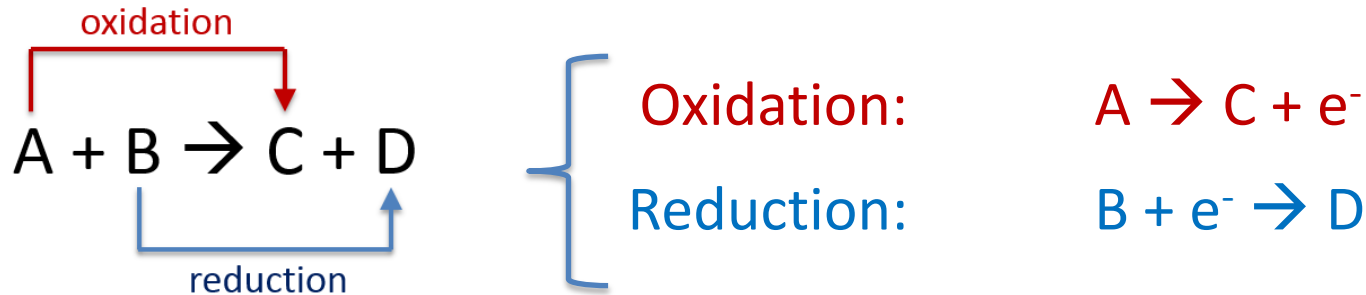
→ Degree of reduction = sum of reduction of all elements in a molecule



# Reaction stoichiometry

## Half reactions

Redox Reaction = oxidation half reaction + reduction half reaction



- Half reactions

1. Electron balance Table (1 & 2)
2. Carbon/nitrogen/sulfur/phosphorus balance
3. Oxygen balance:  $H_2O$
4. Hydrogen balance: protons
5. Charge balance as final check

→ e.g. Acetate to bicarbonate conversion  
(oxidation half reaction)

# Reaction stoichiometry

## Half reactions

Conversion of acetate to bicarbonate\* (oxidation half reaction)

\*  $\text{CO}_2$  at  $\text{pH} = 7 = \text{HCO}_3^-$

1. e- balance (Table 1 & 2):  $\text{C}_2\text{H}_3\text{O}_2^- \rightarrow 8 \text{e}^-$
2. Carbon balance (bicarbonate):  $\text{C}_2\text{H}_3\text{O}_2^- \rightarrow 2 \text{HCO}_3^- + 8 \text{e}^-$
3. Oxygen balance (water):  $\text{C}_2\text{H}_3\text{O}_2^- + 4 \text{H}_2\text{O} \rightarrow 2 \text{HCO}_3^- + 8 \text{e}^-$
4. Hydrogen balance (protons):  $\text{C}_2\text{H}_3\text{O}_2^- + 4 \text{H}_2\text{O} \rightarrow 2 \text{HCO}_3^- + 8 \text{e}^- + 9 \text{H}^+$
5. Charge balance (check):  
$$\underbrace{\text{C}_2\text{H}_3\text{O}_2^- + 4 \text{H}_2\text{O}}_{-1} \rightarrow \underbrace{2 \text{HCO}_3^- + 8 \text{e}^- + 9 \text{H}^+}_{-1}$$

# Reaction stoichiometry

## Half reactions

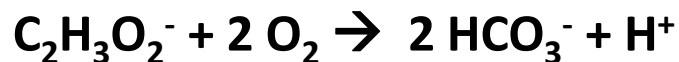
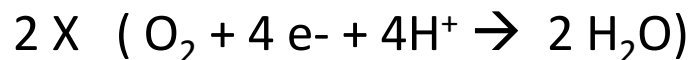
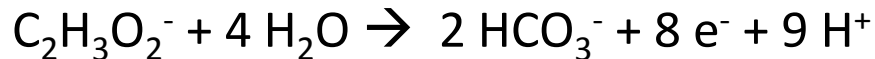
Conversion of oxygen to water (reduction half reaction)

1. e<sup>-</sup> balance:  $O_2 + 4e^- \rightarrow$
2. Oxygen balance (water):  $O_2 + 4e^- \rightarrow 2 H_2O$
3. Hydrogen balance (protons):  $O_2 + 4e^- + 4H^+ \rightarrow 2 H_2O$
4. Charge balance (check):  $\underbrace{O_2 + 4e^- + 4H^+}_0 \rightarrow \underbrace{2 H_2O}_0$

**\*Remember: 1 O<sub>2</sub> gives 4 e<sup>-</sup> then 8 gr O<sub>2</sub>/ e<sup>-</sup> Why?**

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### Complete reaction





# Reaction stoichiometry

All at once

- Complete reactions
    1. Balance the 2 reagents based on the degree of reduction/number of electrons
    2. Carbon/nitrogen/sulfur/phosphorus balance
    3. Oxygen balance:  $\text{H}_2\text{O}$
    4. Hydrogen balance: protons
- Example: Oxidation of acetate

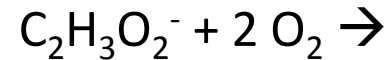
# Reaction stoichiometry

## Adjusting complete reaction directly

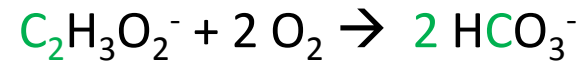
$$\gamma_{\text{C}_2\text{H}_3\text{O}_2^-} = 8 e^- \quad ; \quad \gamma_{\text{O}_2} = -4 e^-$$

$$\text{Acetate:O}_2 = 1:2$$

1. Acetate and oxygen:



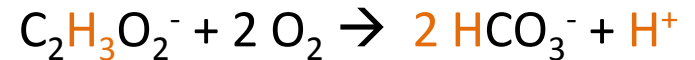
2. Carbon balance with bicarbonate:



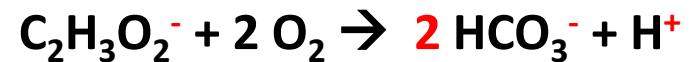
3. Oxygen balance with water:



4. Hydrogen balance (protons):



5. Charge balance (check):



# Exercise 1: nitrification

- Nitrification: conversion of  $\text{NH}_4^+$  to  $\text{NO}_3^-$ 
  - Oxidation reaction
    - Electrons:  $\text{NH}_4^+ \rightarrow 8e^-$
    - Nitrogen:  $\text{NH}_4^+ \rightarrow \text{NO}_3^- + 8e^-$
    - Oxygen:  $\text{NH}_4^+ + 3\text{H}_2\text{O} \rightarrow \text{NO}_3^- + 8e^-$
    - Hydrogen:  $\text{NH}_4^+ + 3\text{H}_2\text{O} \rightarrow \text{NO}_3^- + 10\text{H}^+ + 8e^-$

# Exercise 1: nitrification

– Reduction reaction: reduction of  $O_2$  to  $H_2O$

- Electrons:  $O_2 + 4e^- \rightarrow H_2O$
- Oxygen:  $O_2 + 4e^- \rightarrow 2 H_2O$
- Hydrogen:  $O_2 + 4e^- + 4H^+ \rightarrow 2 H_2O$

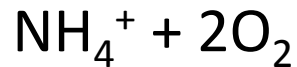
➔ Complete reaction = sum of oxidation and reduction

Or...

# Exercise 1: nitrification

- Complete reaction:

- Electrons: ammonium (8), oxygen (-4)



- Nitrogen:  $\text{NH}_4^+ + 2\text{O}_2 \rightarrow \text{NO}_3^-$

- Oxygen:  $\text{NH}_4^+ + 2\text{O}_2 \rightarrow \text{NO}_3^- + \text{H}_2\text{O}$

- Hydrogen:  $\text{NH}_4^+ + 2\text{O}_2 \rightarrow \text{NO}_3^- + \text{H}_2\text{O} + 2\text{H}^+$

# Calculation of the COD of an organic molecule

- See also theory course notes p. 12
- Steps:
  - Calculate the degree of reduction of the molecule
  - Multiply by 8 (8 g per electron of O<sub>2</sub>)
  - Divide by the molecular weight:  
→ Result = g COD/ g substrate

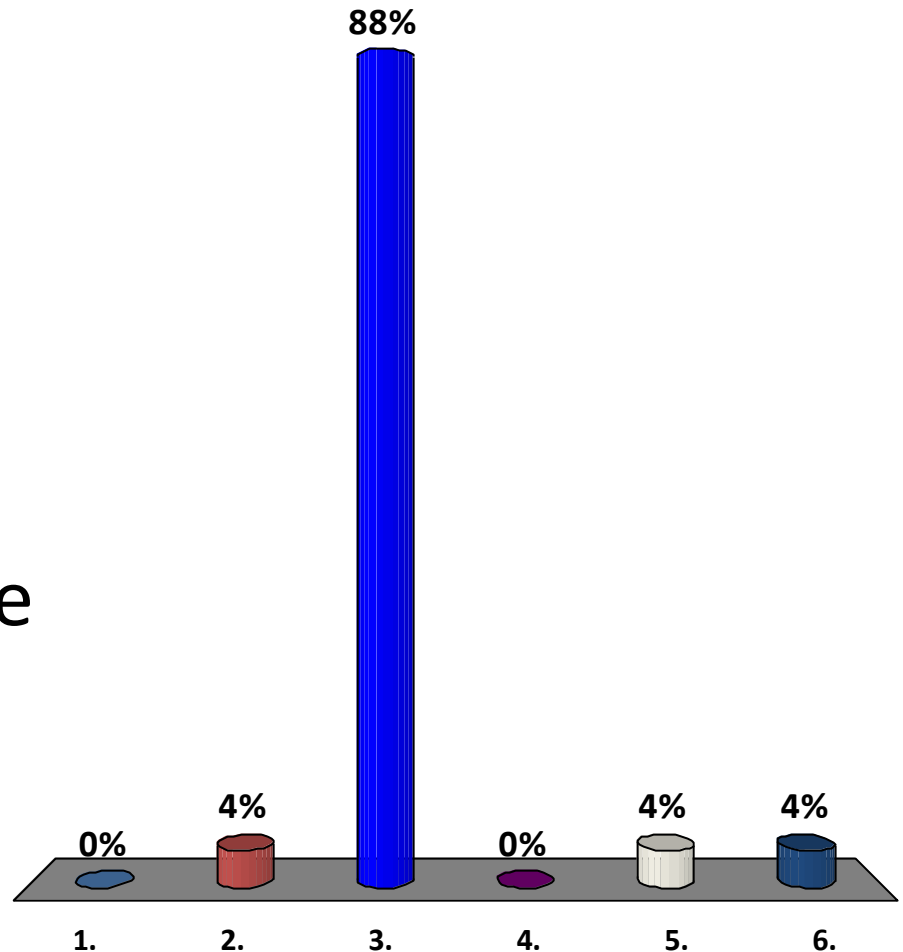
**Example: metanol (CH<sub>4</sub>O)**

**degree of reduction = 6 e<sup>-</sup>**

$$\text{COD} = 6 \text{ e}^- \times 8 \text{ grO}_2/\text{e}^- = 48 \text{ gCOD/mol CH}_4\text{O} = 1.5 \text{ gCOD/gCH}_4\text{O}$$

# Exercise 2: Calculate the COD of ethanol (g COD/g ethanol)

1. 1,39 g COD/g ethanol
2. 96 g COD/g ethanol
- ✓ 3. 2,09 g COD/g ethanol
4. 2,29 g COD/g ethanol
5. I don't know?
6. My answer is not here



# Solution

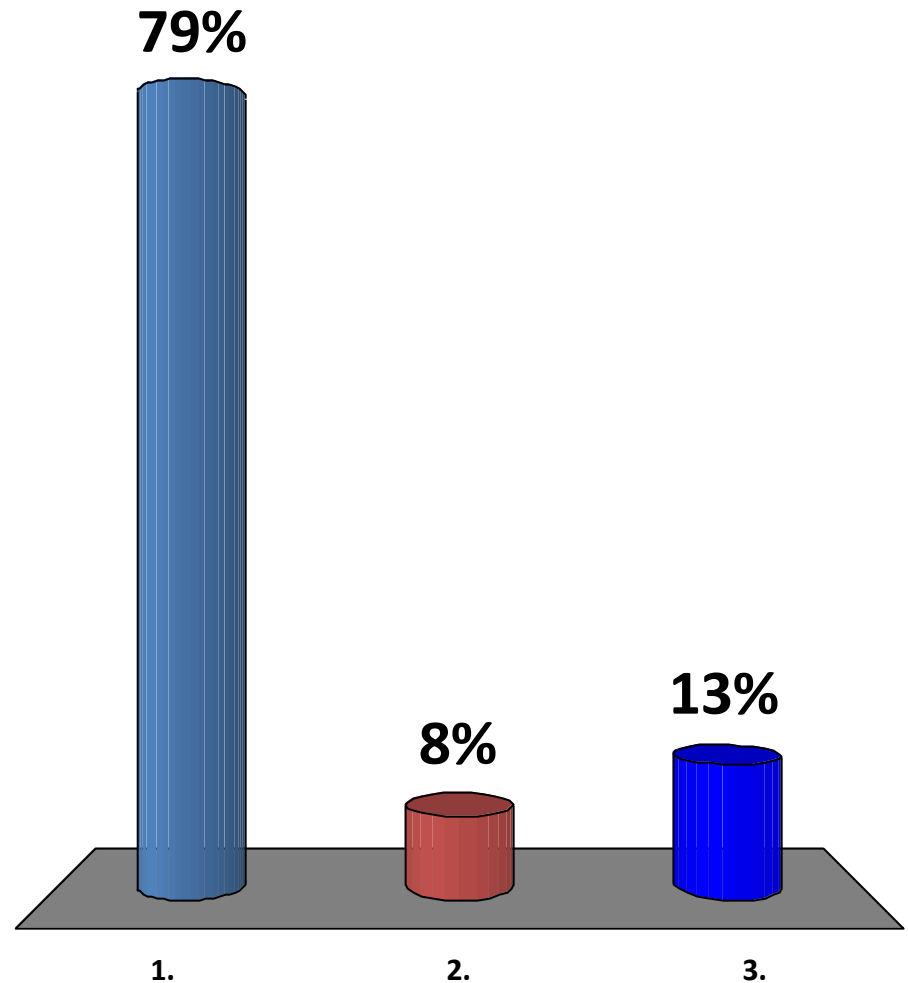
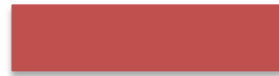
- Ethanol
  - Electrons:  $2 \times (+4) + 6 \times (+1) + 1 \times (-2)$   
 $= 12 \text{ mol e}^-/\text{mol ethanol}$
  - COD =  $12 \times 8 = 96 \text{ g COD/mol ethanol}$
  - COD/weight =  $\frac{96 \text{ g COD/mol}}{46 \text{ g ethanol/mol}}$   
 $= 2,09 \text{ g COD/ g ethanol}$
- Acetate: 1,06 g COD/g
- Glucose: 1,06 g COD/g

• You need to know the formula of simple organic molecules:  
VFAs, glucose, fructose, lower alcohols, nitrate, nitrite, sulfide, sulfate, etc.



# Do you understand the concept of COD calculations with the GRS?

1. Yes
2. No
3. I'm not sure



# COD calculations (reminder)

- Steps:
  - Calculate the degree of reduction of the molecule  
 $\gamma = \text{mol } e^- / \text{mol substrate}$
  - Multiply by 8 (8 g per electron of O<sub>2</sub>)  
→ g O<sub>2</sub> / mol substrate
  - Divide by the molecular weight:  
→ Result = g COD/ g substrate

**EXAM!!!!!!**

# Gibbs energy per electron

$$\Delta G_r'^0 = \sum (v_x \cdot \gamma_x \cdot \Delta G_{e,x})$$

$v_x > 0$  for products

$v_x < 0$  for reagents

$v_x$  = stoichiometric coef.

$\gamma_x$  = degree of reduction

$\Delta G_{e,x}$  = Gibbs free energy of formation (kJ/mol  $e^-$ )

- $\Delta G_r$  : Gibbs free energy of reaction
  - Calculated with conventional values, after production with the reference substances (bicarbonate, nitrate, ...)
- $\Delta G_e$  : Gibbs free energy of formation per electron
  - Divide  $\Delta G_r$  by the degree of reduction ( $\gamma$ )

# Simplifying thermodynamical reactions

- Reaction potentials can be calculated accordingly (e.g. for microbial fuel cells)

$$\Delta E^{0'} = -\frac{\Delta G_r^{0'}}{nF} \quad \text{OR} \quad \Delta E^{0'} = -\frac{\Delta Ge^{0'}}{nF}$$

$$E^{0'} > 0 \quad \img alt="thumbs up icon" data-bbox="235 555 285 653"/> \quad \Delta G'^0 < 0$$

$$E^{0'} < 0 \quad \img alt="prohibited sign icon" data-bbox="225 722 295 810"/> \quad \Delta G'^0 > 0$$

# Concentration effects

- Conditions can differ from 1 atm, 298.15 K, 1 mol/L and pH 7

- If not: correct:

- Temperature
  - Concentration
  - Partial pressure
  - pH
- $$\Delta E_r = \Delta E_r^{0'} - \frac{RT}{nF} \ln(\Pi) + \frac{RT}{nF} \ln \left( \left[ \frac{10^{-pH}}{10^{-7}} \right]^{v_H} \right)$$

➔ Part 1.4 + exercise 3 (@home)

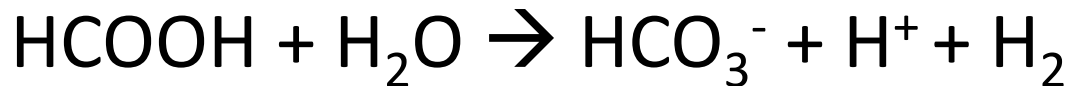
# Excercise 3

The reaction  $\text{HCOOH} \rightarrow \text{H}_2 + \text{CO}_2$  is not thermodynamically favourable. Prove this by calculating the  $\Delta G_r$ . However, it is possible to microbially produce hydrogen from formate if hydrogen is consumed by another species, i.e. the methanogens:  $4\text{H}_2 + \text{CO}_2 \rightarrow \text{CH}_4 + 2\text{H}_2\text{O}$ .

Calculate the new  $\Delta G_r$  assuming that the methanogens keep the concentration of  $\text{H}_2$  [ $\text{H}_2$ ] at  $10^{-5}$  atm. The reaction occurs at pH 7 and temperature  $25^\circ\text{C}$ .

- Solution: 2 ways to calculate

1°



$\gamma$  for  $\text{H}_2\text{O}$ ,  $\text{HCO}_3^-$  and  $\text{H}^+$  are 0

$$\Delta G_r^{\circ'} = v_{\text{H}_2} * \gamma_{\text{H}_2} * \Delta G_{\text{H}_2} - v_{\text{HCOOH}} * \gamma_{\text{HCOOH}} * \Delta G_{\text{HCOOH}}$$

$$\Delta G_r^{\circ'} = 80 - 78 = 1.368 \text{ kJ/mol} \rightarrow \text{does not allow growth}$$

If concentration  $[\text{H}_2]$  is kept low (at  $10^{-5}$  atm):

(at pH 7)

$\rightarrow$  allows growth

1°

If concentration  $[H_2]$  is kept low (at  $10^{-5}$  atm):  
(at pH 7)

$$\Delta G_r = \Delta G_r^{0'} + RT \ln(\Pi) - RT \ln \left( \left[ \frac{10^{-pH}}{10^{-7}} \right]^{v_H} \right) = \Delta G_r^{0'} + RT \ln(\Pi)$$

$$\Pi = \frac{a_C^{v_C} a_D^{v_D}}{a_A^{v_A} a_B^{v_B}} \approx \frac{[C]^{v_C} [D]^{v_D}}{[A]^{v_A} [B]^{v_B}} = \frac{[HCO_3^-][H_2]}{[HCOOH]} = \frac{1 * (10^{-5})}{1} = 10^{-5}$$

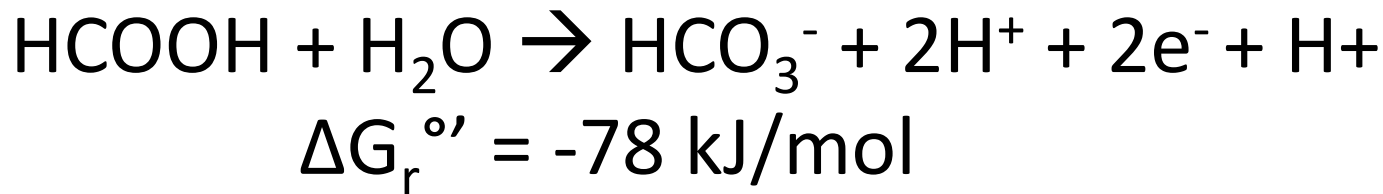
$$\Delta G_r = 2 \text{ kJ/mol} + RT \ln(\Pi) = 1.368 \text{ kJ/mol} + 8.314 \frac{\text{m}^2 \text{ kg}}{1000 \text{ s}^2 \text{ K mol}} (25 + 273) \text{ K} * \ln(10^{-5})$$

$$\approx -27 \text{ kJ/mol}$$

→ allows growth



2°



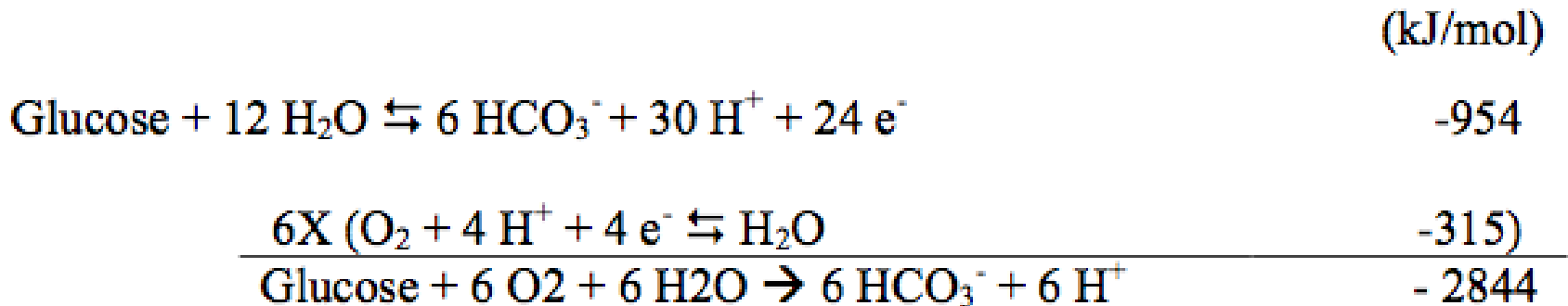
$$\Delta G_r = \Delta G_r^{\circ'} + RT \ln\left(\frac{1}{H_2}\right)$$

$$\Delta G_r = \Delta G_r^{\circ'} + RT * 2.303 * (\log[H_2] - 2\log[1]) = \Delta G_r^{\circ'} + RT * 2.303 * (-5) \text{ of } \Delta G_r^{\circ'} + RT * \ln(10^{-5}) \\ = 51.476 \text{ kJ/mol}$$

$$\rightarrow 51.476 - 78 = \underline{\underline{-27 \text{ kJ/mol}}}$$

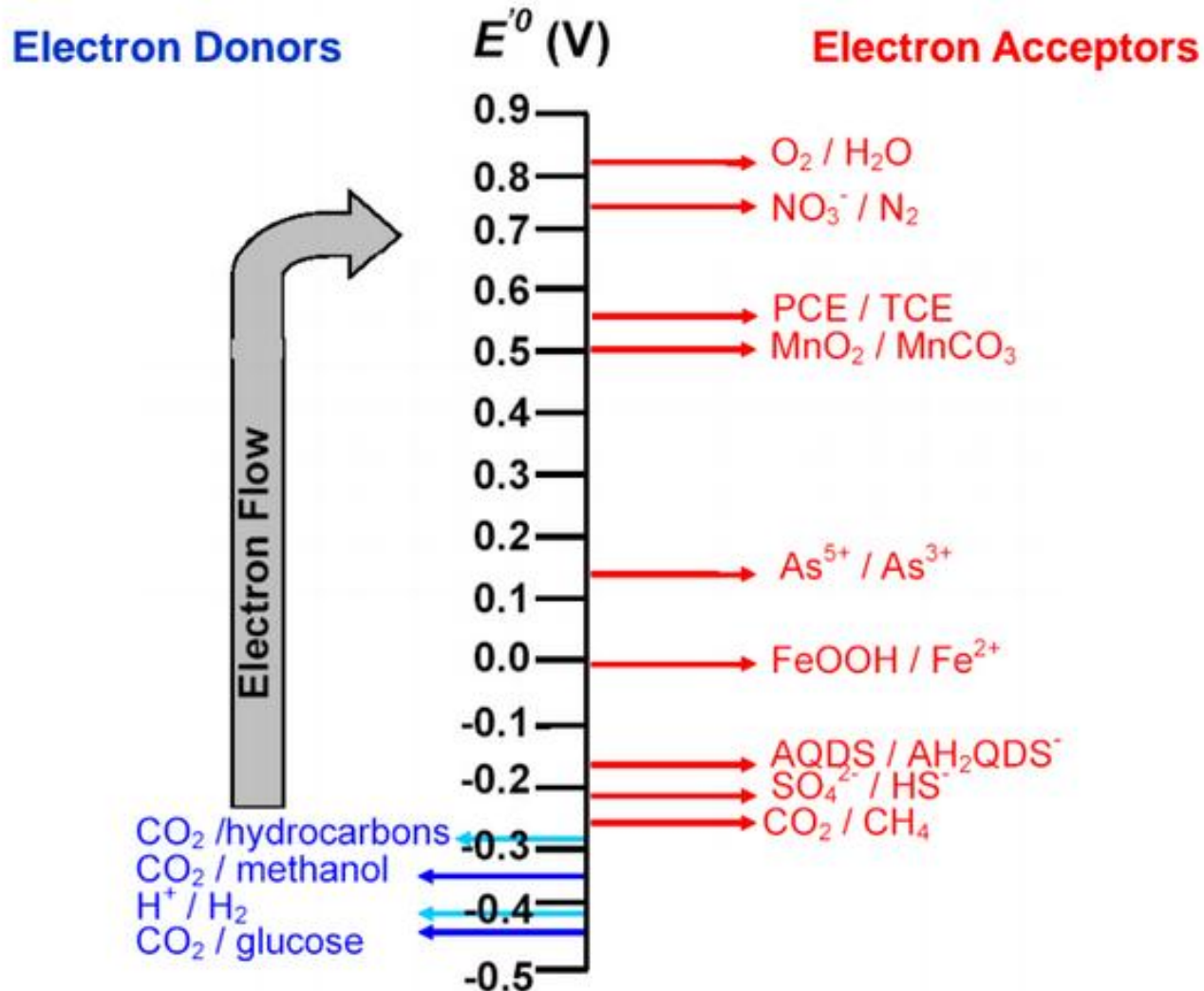
# Stoichiometry in WWTP

- Organic matter removal
  - Domestic waste:  $C_{10}H_{19}O_3N$ ,  
glucose as simplification ( $C_6H_{12}O_6$ )
  - Aerobic:  $O_2$  as electron acceptor



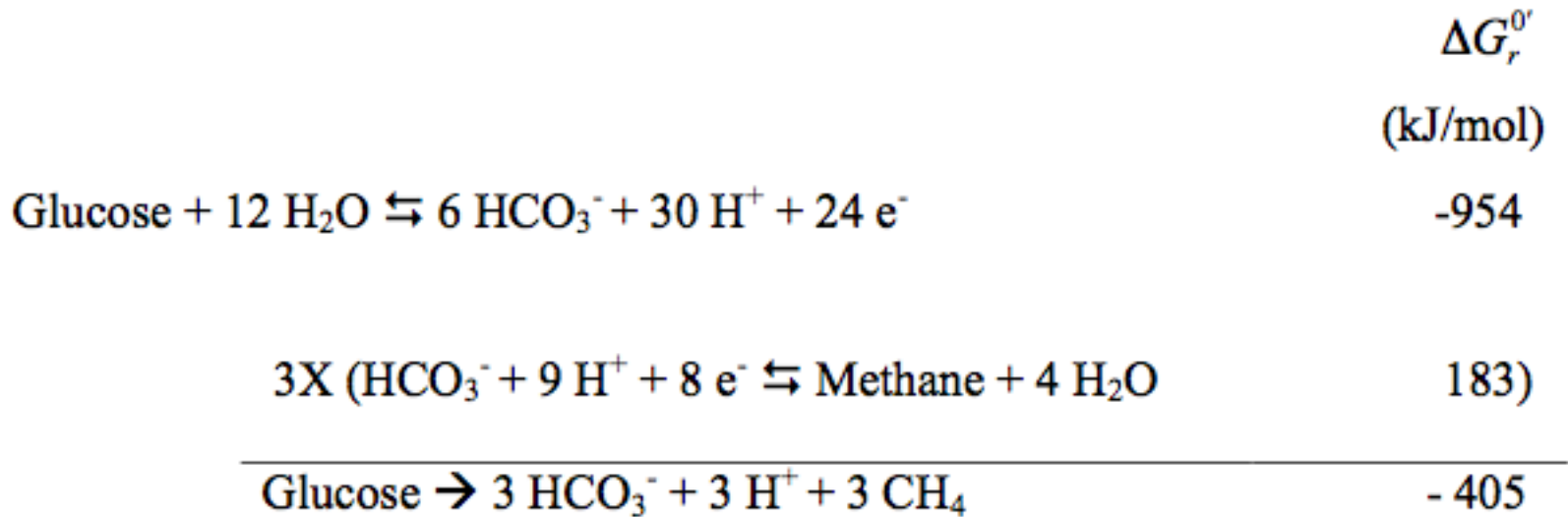
# Stoichiometry in WWTP

In the absence of  $O_2$  some bacteria can use different e- acceptors



# Stoichiometry in WWTP

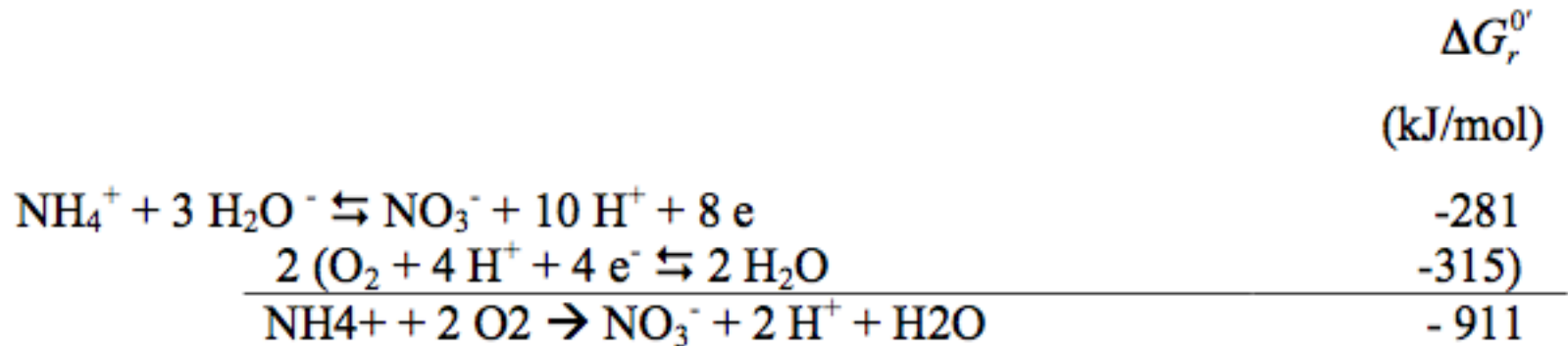
- Anoxic/Anaerobic: No oxygen as electron acceptor; thermodynamically less favourable
- $\text{HCO}_3^-$  as electron acceptor (e.g. anerobic digestion)



- Other electron acceptors: see removal of nutrients

# Stoichiometry in WWTP

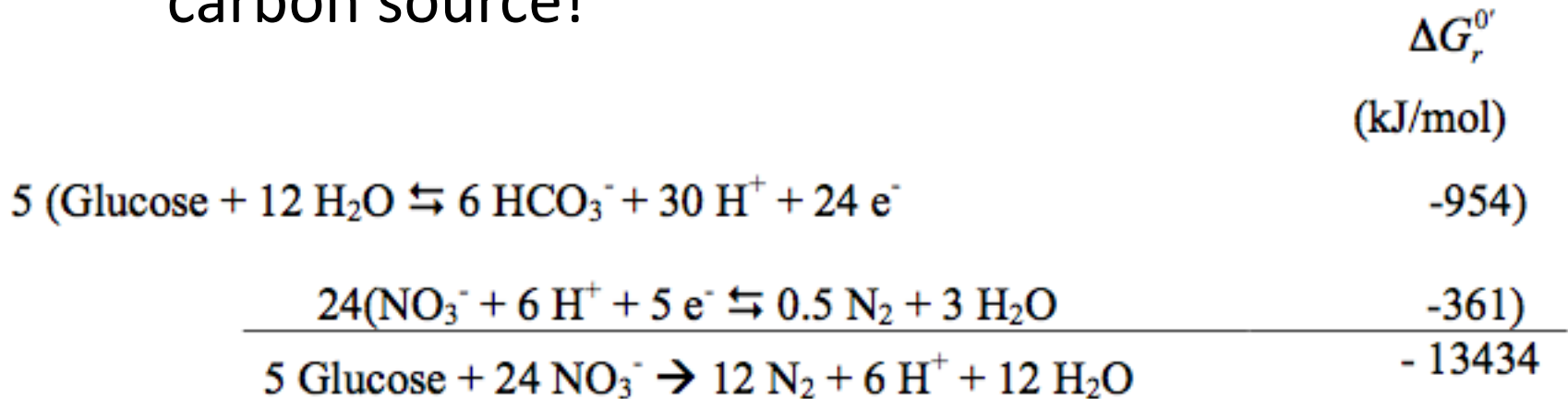
- Nitrogen removal
  - Nitrification: autotrophic bacteria



How much  $\text{O}_2$  needed per gram of N?

# Stoichiometry in WWTP

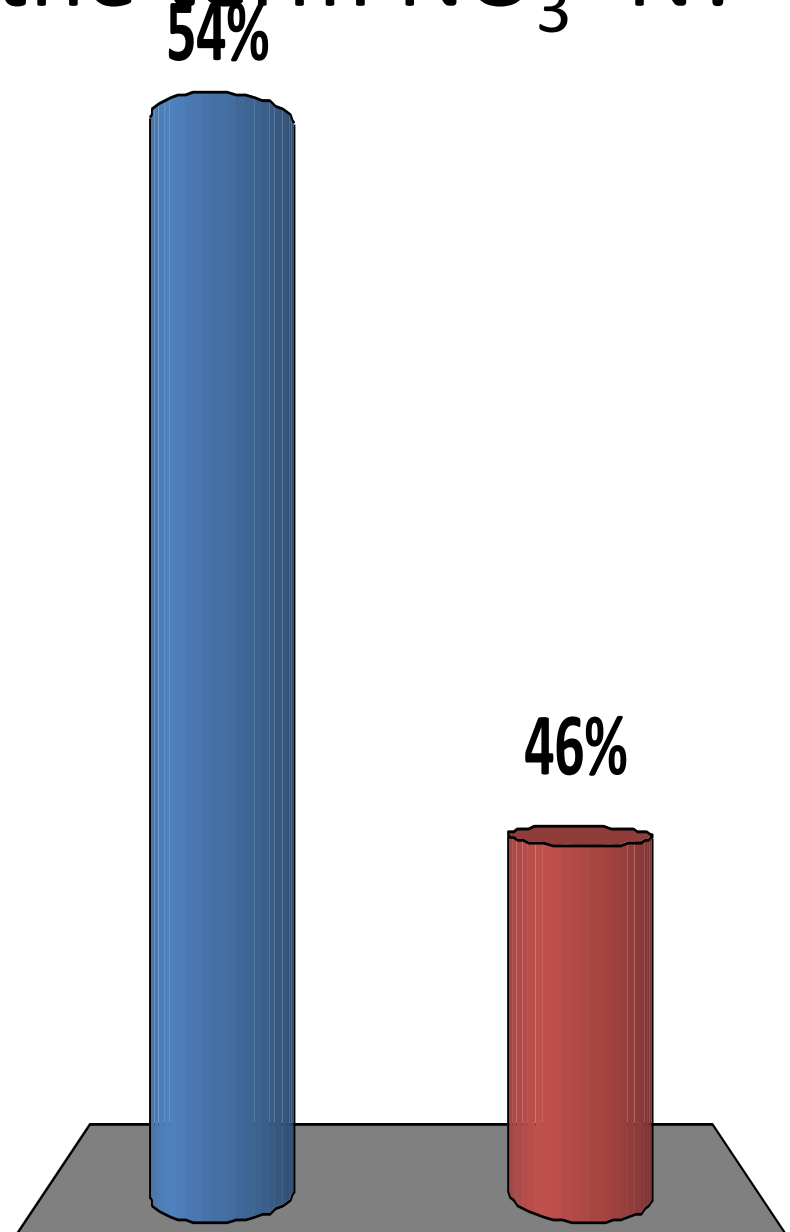
- Nitrogen removal
  - Denitrification: heterotrophic bacteria: need a carbon source!

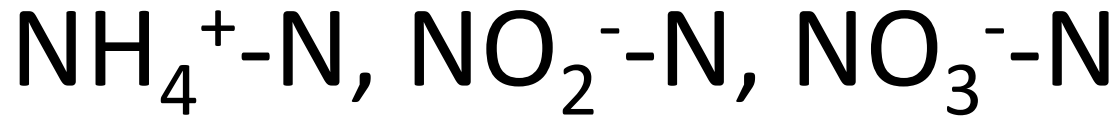


How much glucose needed per gram of N?

# Do you understand the term $\text{NO}_3^-$ -N?

1. Yes
2. No



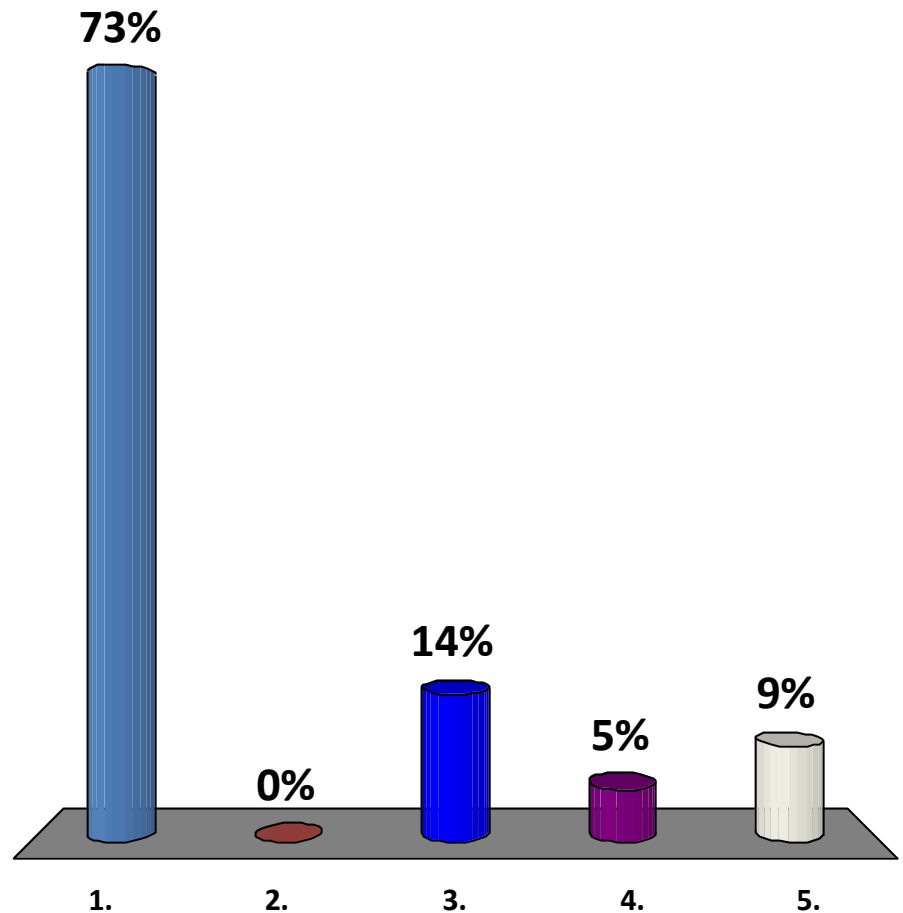


- Nitrogen
- Origin of nitrogen source is added
- e.g.  $\text{NH}_4^+-\text{N}$ ,  $\text{NO}_3^--\text{N}$
- Only take molecular mass of N into account, no other elements
- Also for other compounds:  $\text{SO}_4^{2-}-\text{S}$  ,  $\text{PO}_4^{3-}-\text{P}$



# How much glucose is needed per g N removed (through denitrification)

- ✓ 1. 2,68 g
- 2. 0,60 g
- 3. 0,21 g
- 4. 12,86 g
- 5. ?

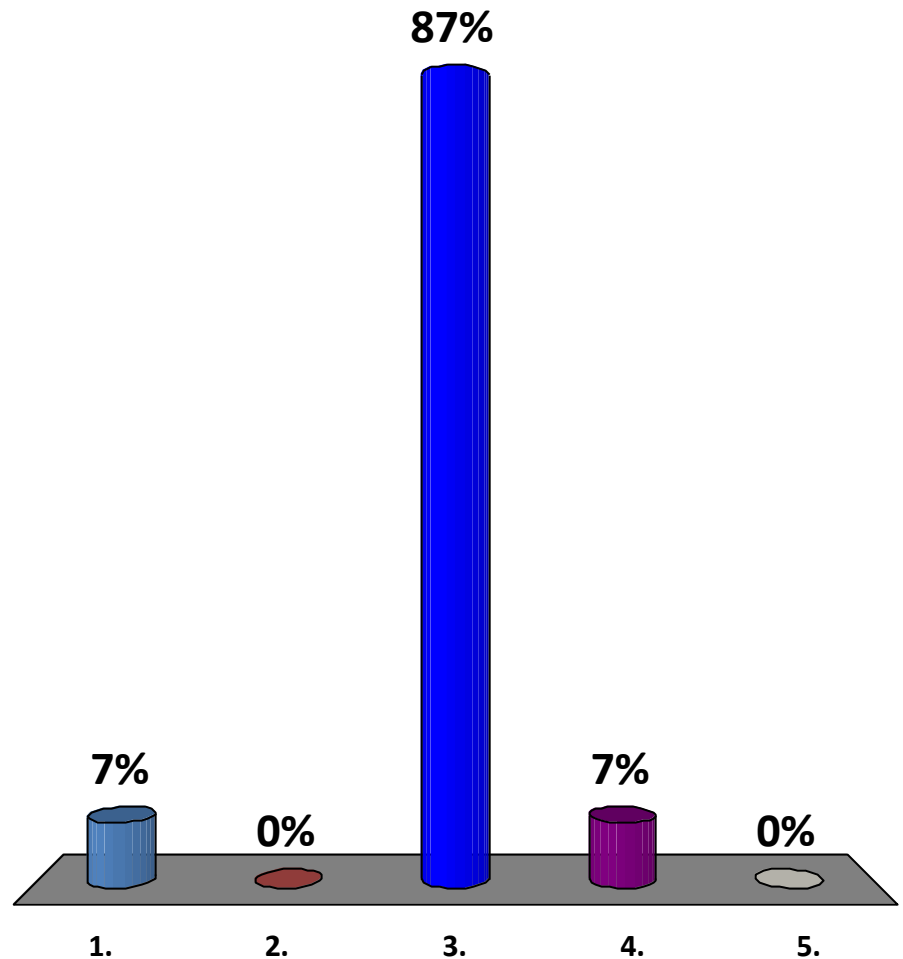


# Exercise 4a

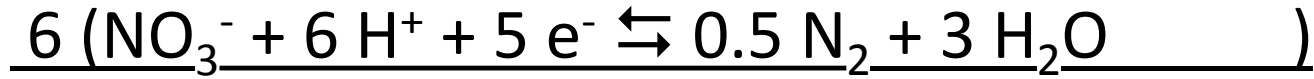
5 mol glucose per 24 mol N = 900 g glucose per  
336 g N = 2,68 g glucose per g N

# Calculate the price of methanol per kg $\text{NO}_3^-$ -N removed

1. 0,80 €/kg N
2. 0,15 €/kg N
- ✓ 3. 0,67 €/kg N
4. 1,00 €/kg N
5. ?



# Exercise 4b



→ 5 mol methanol per 6 mol N = 160 g methanol per 84 g N = 1,90 g methanol per g N

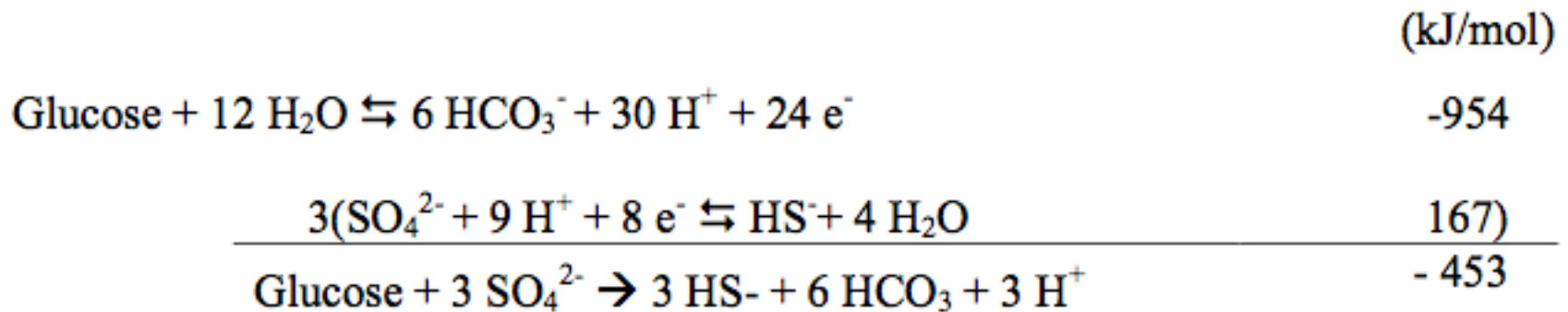
→ **€ 0,67 per kg N removed**

→ Glucose: € 0,8 per kg N removed

→ Methanol = cheaper than glucose

# Stoichiometry in WWTP

- Sulfate reduction: reduction to sulfide
- In sewer systems: corrosion due to formation of  $\text{H}_2\text{S}$   $\rightarrow$  to be avoided!



# Biomass production

- Previous reactions = Catabolic
- Need also to include anabolic reactions → bacteria use part of the electron donor (C) for biomass synthesis (also  $\text{NH}_4^+$ -N)
  - Autotrophic bacteria:  $\text{HCO}_3^-$
  - Heterotrophic bacteria: Organic Carbon ( $e^-$  donor)

# Biomass production

**Table 1: Mole fractions of the electron donor used for biomass synthesis in the different reactions**

Electron donor	Electron acceptor	Fraction for synthesis
Glucose	O <sub>2</sub>	0.7
Glucose	HCO <sub>3</sub> <sup>-</sup>	0.05
Methanol	NO <sub>3</sub> <sup>-</sup>	0.5
NH <sub>4</sub> <sup>+</sup>	O <sub>2</sub>	0.1

- Less biomass production in the anaerobic system = big advantage!
- Less sludge production = less sludge to be removed

# Wrap-up

At the end of the semester you can:

- Calculate the COD of simple molecules used in WWTP
- Determine the energy gain/input for biochemical reactions using the growth reference system (GRS)
- Comment on sludge production under various conditions in WWTP



# When do we meet

Oct 29 <sup>th</sup>	Stoichiometry	
Nov 5 <sup>th</sup>	Removal of organic matter Sludge treatment	13h00 Denduyver
Nov 12 <sup>th</sup>	Nutrient removal Sedimentation	13h00 Denduyver
Dec 2 <sup>nd</sup>	Combined exercise // Overall design 8h30 HILO auditorium A	

*Filters, aeration, fluidized-bed, oxidation ditch: additional information*